the change in the parameters when the soil withstands tensile stresses $\sigma^{\circ} = -0.2$. Loss of continuity was not taken into account in formulating the problem.

The solution obtained shows that the laws of wave–barrier interaction depend importantly on both the plastic and the viscous properties of the medium. Viscosity leads to broadening of the reflected and transmitted waves and the barrier load and modifies their profile, at the same time reducing the maxima of the barrier load and acceleration.

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STRESSES IN THE ZONE OF THE WETTING LINE AND THE DYNAMIC RESISTANCE OF THE MENISCUS

B. V. Zheleznyi and A. G. Nikiforov

UDC 532.68

INTRODUCTION

The use of the usual "condition of adhesion" of a liquid to a solid surface in the analysis of the flow in the zone of the line of solid/liquid/gas three-phase contact (LTC) leads to a solution with mathematical singularities at the line of three-phase contact [1]. Remaining within the framework of the continuum mechanics of a liquid, these singularities can be eliminated either by renouncing the condition of adhesion for the zone of the line of three-phase contact or by assuming that the solid surface near the meniscus is covered with a polymolecular (liquid) film, so that the line of three-phase contact, as such, is absent (there is no "wetting line," but only a finite extension of the transitional region between the meniscus and a film of homogeneous thickness). In the latter case, the condition of adhesion can be used.

The problem of the motion of the meniscus with the presence of a liquid film on the wall is formulated in [2, 3]. The difference between the "departing" and "arriving" menisci is connected with the fact that, in the first case, the mean thickness h_* of the film (remaining on the wall) is determined by the velocity of the meniscus v, while, in the second case, the thickness h_* of the film ahead of the meniscus can be given arbitrarily. In the case of the presence of an additional independent variable (h_*), the case of an arriving meniscus is mathematically more complicated.

The principal practical problem, solvable for a departing meniscus, is to find the dependence $h_*(v)$, while, for an arriving meniscus, it is to find the effective hydrodynamic resistance. The first problem was solved in [2], taking account of the specific thermodynamic and rheological properties of "thin" films, while the second problem was solved in [3] for the case of rather "thick" films having the properties of a volumetric liquid. In the latter

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case, the dependence $\xi(v)$ was found, where ξ is the relative radius of curvature of the meniscus.

The present article gives the results of an analysis of the flow in the transitional zone from the departing meniscus to a "thin" film, made within the framework of a method described in [2]. Two questions are discussed: the distribution of the stresses in the transitional zone and the effective dynamic resistance of the meniscus. The notation and terminology introduced in [2] are used here also.

§1. In the approximation of flat films under consideration, two kinds of stresses, arising in the flow, are significant: the pressure gradient G in the direction of the flow (the x axis) and the tangential stress τ_0 at the wall. The distributions of both stresses along the x axis can be found if the profile of the transitional zone from the meniscus to the film is known. For the models of films considered in [2], the value of G can be represented in the form

$$G=\left[\frac{0.8\sigma}{R^{3/2}\sqrt{h_0}}\right]\left\{\frac{1}{C^2(\alpha,\beta)\sqrt{W}}\frac{y-1}{y^3}\left(1+\frac{\beta}{y}\right)^{\nu}\right\}, \tag{1.1}$$

where $\nu = 1$ for a stable-film model and $\nu = -1$ for an unstable-film model.

In [1], the problem was solved for a capillary of radius R; however, the statement and solution of the problme remain valid also for systems of more complex geometry if the meniscus moves in the direction x, perpendicular to the wetting line, and the condition $R_s \gg R_m$ holds. Here R_s and R_m are, respectively, the radii of curvature of the solid surface and the meniscus in the plane of a meridional cross section (perpendicular to the wetting line). By R, in the general case, there must be understood the distance from the solid surface to the center of a circle (of radius R_m), inscribed in the central part of the meridional cross section of the meniscus. Between the meniscus and the film there is a transitional zone where the radius of curvature varies smoothly from $\sim R_s$ to T_m.

In the square brackets in (1.1) there is separated out the dimensional part, determined only by the value of R and the equilibrium parameters of the system. The dependence of G on the velocity is included in the dimensionless part (in curly brackets), determined by the value of the parameters α and β and the concrete model of the film (the latter determines the value of W and the value of the coefficient $C(\alpha, \beta)$ for given values of α and β; the method of obtaining W is given in [2]).

The value of G is constant in a given cross section of a flat film (perpendicular to the x axis); the value of the tangential stress τ at a distance z from the "free" surface of the film (at which it is assumed that there is no tangential stress) is equal to $\tau = Gz$. The maximal value of $\tau = \tau_0$ in a given cross section is attained at the wall and is equal to Gh, where h is the thickness of the film in the given cross section. Analogously to (1.1), we have

$$\tau_0=\left[\frac{1.25\sigma\sqrt{h_0}}{R^{3/2}}\right]\left\{\frac{\sqrt{W}}{C(\alpha,\beta)}\frac{y-1}{y^2}\left(1+\frac{\beta}{y}\right)^{\nu}\right\}, \tag{1.2}$$

where the square and curly brackets include, respectively, the dimensional and dimensionless parts of the expression.

Expressions (1.1) and (1.2) do not contain x explicitly; however, in actuality, they express the distribution of the stresses along the x axis, since y is a function of x. From (1.1) and (1.2) it follows that for certain values of y, maximal values (G_m and τ_m) of the stresses G and τ_0 in the transitional zone are attained. Specifically, for $\beta = 0$ (a standard film), G_m is attained for $y = 1.5$ and τ_m for $y = 2$. Examples of the distributions of G and τ_0 along the x axis for a standard stable film with n = 3 are given in Figs. 1 and 2, which present the dimensionless parts (notation G_* and τ_*) of expressions (1.1) and (1.2) for three values of the dimensionless velocity: V = 0.1, 1, and 10, curves 1-3, respectively). These distributions were obtained as a result of an analysis of integral curves of y(x) for the corresponding partial solutions.

For convenience in comparison of the distributions, the scale along the axis of abscissas in Figs. 1-4 was taken differently from the scale of the variable x used in [2]; the dimensionless coordinate $X = (0.643R/h_0)^{1/2}l/h_*$; the start of the reckoning along the X axis was taken to correspond to the level $y = 2$.

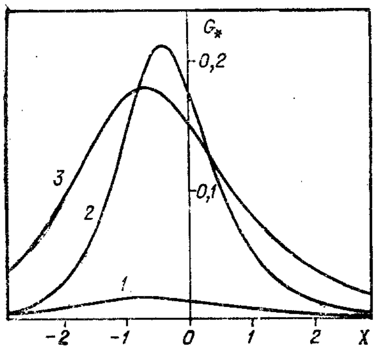

Fig. 1

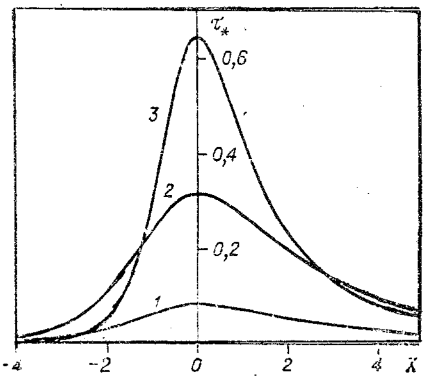

Fig. 2

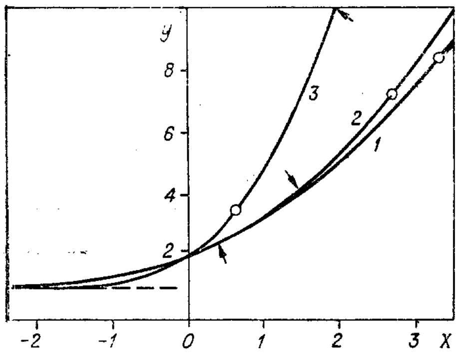

Fig. 3

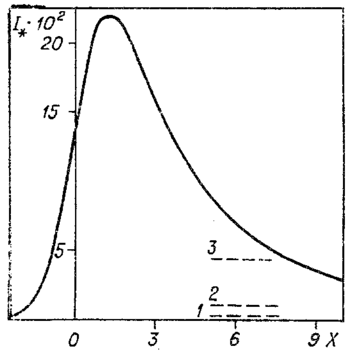

Fig. 4

Figure 3 gives the profile of the transitional zone from the meniscus to the film for the model under consideration for the three values of the velocity given above. The actual "flatness" of the film (dh/dl) can be evaluated from the curve in Fig. 3 using the relationship $dh/dl = \sqrt{h_0/0.643R} \cdot dy/dX$. Points corresponding to a value of $dy/dX = 3$ are denoted in Fig. 3 by a circle. This value of dy/dX corresponds to a value dh/dl on the order of 0.1 in a capillary of radius ~10 μm and, correspondingly, to lower values of dh/dl and wider capillaries. The arrows in Fig. 3 indicate the level at which the second derivative y'' attains 99% of its limiting value. From a comparison between Fig. 3 and Figs. 1 and 2, it follows that the maximal stresses develop precisely in the region of flatness; the approximation of a flat film is the better, the less the value of V and the greater the value of R.

By G_P and τ_P we denote the pressure gradient and the stress at the wall for unperturbed Poiseuille flow in a cylindrical capillary at some distance from the meniscus. The ratios of the stresses in the transitional zone corresponding to these stresses are

$$G/G_P = 0.155(R/h_0)^2 G_*/V; \tag{1.3}$$

$$\tau_0/\tau_P = 0.484R\tau_*/h_0V, \tag{1.4}$$

where G_* and τ_* are the dimensionless parts of the expressions (1.1) and (1.2). From (1.3) and (1.4) it follows that the stresses developed in the transitional zone can exceed by many orders of magnitude the stresses arising in the flow of liquid far from the meniscus.

Figure 5 gives the dependence of V on the dimensionless parts (G_{*m} — 1 and τ_{*m} — 2) of the maximal stresses (G_m and τ_m) in the transitional zone for the same model of a stable film, from which it can be seen that, with a rise in the velocity of the meniscus, the value of τ_m rises monotonically, while the value of G_m passes through a flat maximum. As $V \to 0$, in the case of a stable film the profile of a thermodynamically equilibrium meniscus is attained (it practically coincides with the profile for V = 0.1 in Fig. 3); under these circumstances $G \to 0$ and $\tau \to 0$, which can be seen in Figs. 1 and 2.

The case of an unstable film is qualitatively different. Here a thermodynamically equilibrium meniscus is also attained as $V \to 0$ and corresponds to zero stresses. However, this takes place for velocities less than the critical wetting velocity (V_0), where a finite departing contact angle is attained and the surface behind the meniscus remains "dry." In this case the approximation of a flat film becomes incorrect. With an approach to V_0 from the side of large velocities, the stresses in the flat part of an unstable film rise unboundedly, which follows from (1.1) and (1.2): $G \to \infty$ and $\tau_0 \to \infty$ as $W \to W_0$, since $C(\alpha, \beta) \to 0$.

An unbounded rise of the stresses in the zone of the line of three-phase contact is generally characteristic for motion over a "dry" surface [1]; however, within the framework of the effect under consideration, this effect is inseparably connected with the special thermodynamic characteristics of a thin unstable film. We note that the assumption of the possibility of slip of the liquid along a lyophobic solid surface in this case does not eliminate the effect of a catastrophic rise in the stresses. In turn, large tangential stresses at the wall in the zone of the line of three-phase contact should lead to a clear effect of slipping of the liquid with the motion of an arriving meniscus over a lyophobic surface; an analysis of existing literature on the critical wetting velocity confirms this point of view. We note that under ordinary flow conditions, for example, for Poiseuille flow having a mean velocity equal to the velocity of the meniscus, the slip may be completely inappreciable. This is connected, above all, with the smallness of the tangential stresses at the wall for Poiseuille flow far from the meniscus.

§2. The effective or additional dynamic resistance of the meniscus (p) can be determined in different ways. The usual method of expressing p in terms of the contact angle θ is convenient for $\theta > 0$ and, in practice, is sufficiently exact, since appreciable deviations of the curvature of the meniscus from the value corresponding to its central part (in the approximation of the force of gravity) at ordinary velocities occur only in a region having an extension of the order of 10^{-5} cm near the line of three-phase contact [4]. More considerable distortions of the form of the meniscus at a lyophobic surface are possible for large velocities where $\theta \to 0$ or $\theta \to \pi$. For a cylindrical capillary, this method for determining p gives

$$p = 2\sigma(\cos\theta - \cos\theta_0)/R, \tag{2.1}$$

where θ and θ_0 are, respectively, the dynamic and equilibrium contact angles; p is referred to unit area of the cross section of the capillary.

The method in question can be generalized for the case of a lyophilic surface ($\theta_0 \approx 0$) if $\cos\theta$ in (2.1) is replaced by the quantity $\xi \equiv R/R_m$:

$$p = 2\sigma(\xi - \xi_0)/R, \tag{2.2}$$

where ξ_0 corresponds to a thermodynamically equilibrium meniscus. For a lyophilic surface, $\xi_0 \geqslant 1$, and for a lyophobic surface, $\xi_0 \equiv \cos\theta_0 < 1$. For a departing meniscus it is possible that $\xi > 1$ on both lyophilic and lyophobic surface. As in the case of (2.1), expression (2.2) is sufficiently exact under the condition that the linear dimensions of the transitional zone in which the main change in the curvature of the meniscus occurs are small in comparison with R_m. In this case, the pressure in the liquid directly at the (moving) meniscus can be considered practically hydrostatic, corresponding to R_m, and formulas (2.1) and (2.2) describe the deviation of this pressure from the thermodynamically equilibrium value.

For the model of a stable film, discussed in [2], the following relationship holds:

$$\xi_0 = 1 + \frac{h_0}{R_m}\frac{n}{n-1} \approx 1 + \frac{h_0}{R}\frac{n}{n-1}. \tag{2.3}$$

With respect to the dependence $\xi(v)$, on the basis of [2, 3] it can be represented in the form

$$\xi = 1 + \frac{h_0}{0.643R} f(V, \gamma), \tag{2.4}$$

where $f(V, \gamma)$ is some function of the dimensionless velocity V and the parameter γ, determined by the concrete model of the film. This function is connected with the coefficients $C_{1\infty}$ and $C_{2\infty}$, determined in accordance with [3], by the relationship $f(V, \gamma) = C_\infty(\alpha, \beta)W$, where $C_\infty(\alpha, \beta) = C_{1\infty}(\alpha, \beta)C_{2\infty}(\alpha, \beta)$. The manner of writing $C(\alpha, \beta)$ means that each of the above-listed coefficients is a function of the parameters α and β. Tables of the coefficients $C_{1\infty}(\alpha, \beta)$ for a departing meniscus, calculated for different models of the films, are given in [2] [the designation $C(\alpha, \beta) \equiv C_{1\infty}(\alpha, \beta)$ is used there]. The coefficients $C_{2\infty}(\alpha, \beta)$ and $C_\infty(\alpha, \beta)$ were not calculated in [2]; in [3], they were calculated for an arriving meniscus within the framework of a model of an "ideal" film.

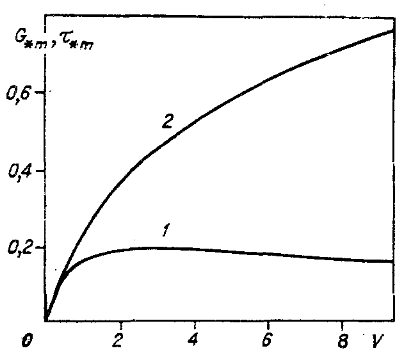

Fig. 5

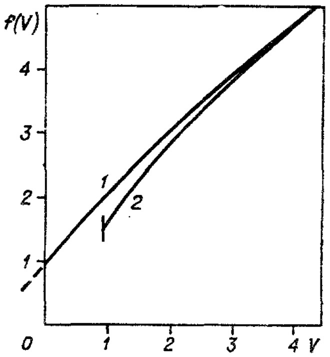

Fig. 6

Figure 6 gives curves of the function f(V) for a departing meniscus, calculated using the models of [2] of stable and unstable "standard" ($\gamma = 0$) films for n = 3. For a stable film (curve 1), the value of f(v) for V = 0 corresponds to ξ_0. For small values of V the dependence f(V) is almost linear; for a stable film it can be approximated by the formula $f(V) \approx 0.965 + 1.1V$. Together with (2.2)-(2.4), this gives

$$p \approx 3.4 h_0 \sigma V / R^2.$$

In the case of an unstable film (curve 2), the dependence f(V) has a discontinuity with a critical wetting velocity (V_0) for $V > V_0$ and, as in the case of a stable film, $\xi > 1$.

We introduce the concept of the "relative resistance of the meniscus" (L_*), defined in the following manner. Let L be the length of a section of a cylindrical capillary in which, with Poiseuille flow (far from the meniscus), the pressure drop is numerically equal to p. If the mean velocity of the flow is equal to v, then $L = pR^2 8\eta_0 v$. We define $L_* \equiv L/R$. Then, for a departing meniscus, within the framework of a model of a stable film, it holds that

$$L_* = 0.75\sqrt{0.643R/h_0}\,[f(V, \gamma) - 0.965]/V.$$

Using the above approximation for a model of a standard stable film with n = 3, for small velocities we obtain

$$L_* \approx 0.66(R/h_0)^{1/2} = 1.08R^{1/3}(\sigma/A)^{1/6}. \tag{2.5}$$

It is worthy of note that, according to (2.5), L_* does not depend on viscosity forces, but is determined only by the thermodynamic special characteristics of a thin film. This is a consequence of the linear approximation of the function f(V) for the model in question. The dimensionless complex $M_0 \equiv h_0/R$ entering into (2.5) plays an important role in problems of the type under consideration. An evaluation of L_* in accordance with (2.5) gives $L_* \approx 20$ for R ~ 10 μm. Thus, even in the case of complete wetting, the relative resistance of the meniscus is very considerable. This case can serve as a simple illustration of the so-called "static hysteresis" of the contact angle.

The dashed line in Fig. 6 corresponds to a linear extrapolation of dependence f(V) to the case of an arriving meniscus (V < 0) moving in a film of thickness h_0 (for the same model). In this case formula (2.5) in a limited interval of velocities remains valid also for an arriving meniscus. To find the resistance of an arriving meniscus, moving over a lyophilic surface with an arbitrary value of h_*, we require the solution of a problem of the type of [3] for models of stable films [2].

The above-described method for finding p can be called the "force" model. Another method for finding p (the "energy" method) is based on calculation on the dissipation of energy in the zone of the line of three-phase contact. The value of the dissipation of energy (D_f), arriving in unit time at unit area of the solid surface in the flat part of the film, is equal to $G^2h^3/3\eta_0$, which, together with (1.1), gives for the models under consideration

$$D_f = (0.643)^{1.5}\sigma^2 H^4 M_0^{2.5}(3\eta_0)^{-1} I_*, \tag{2.6}$$

where $I_* = G_*^2 y^3$. At unit length of a cylindrical capillary in the flat region of the film there arrives a dissipation $D_m = 2\pi R D_f$. The dissipation D_p for unperturbed Poiseuille flow far from the meniscus, arriving in unit time at a unit of length of the cylindrical capillary, is equal to $8\pi\eta_0 v^2$, where v is the mean velocity of the flow over the cross section. The ratio D_m/D_p is

$$D_m/D_P = I_* H^3 (7.8\, M_0 V^2)^{-1}. \tag{2.7}$$

The ratio D_m/D_P can be very great, as illustrated in Fig. 4, which gives the distribution of the value of I_*, entering into (2.6) and (2.7), along the X axis, calculated for a model of a standard ($\gamma = 0$) stable film with $n = 3$ and $V = 1$. The same figure gives the corresponding levels of the Poiseuille flow in capillaries with diameters of 10, 1, and 0.1 μm (dashed lines 1-3, respectively). For a comparison with the Poiseuille dissipation D_P, the thermodynamic parameters of thin films of water on quartz were taken in accordance with [5]. From Fig. 4 it can be seen that in wide capillaries, the maximum of the dissipation occurs in the flat part of the film; in the region of the maximum $D_m \gg D_P$. This effect explains the high relative resistance of the meniscus for large values of R following from (2.5).

The dependence $I_*(X)$, illustrated in Fig. 4, was calculated in the approximation of a flat film. For sufficiently great values of y (corresponding to X), this approximation becomes untrue, which explains the apparent intersection of the curve of $I_*(X)$ with a level corresponding to D_P for a capillary of given radius. As has been noted above, the approximation of a flat film is found to be the better, the greater the value of R; this can be seen also in Fig. 4. In sufficiently narrow capillaries ($R \sim 0.01$ μm), the level of the Poiseuille dissipation becomes so high that the maximum on the actual dependence $I_*(X)$ vanishes altogether. In this case the relative resistance of the meniscus is relatively small.

To calculate the effective resistance of the meniscus from the dissipation of energy it is necessary to find the total value of the dissipation D_1 in the zone of the meniscus and to subtract from it the value of D_2, corresponding to the contribution of Poiseuille dissipation. The latter can be determined formally in a different manner; however, here there arises a definite difficulty. We assume that the Poiseuille conditions far from the meniscus are completely formed and that it is possible to determine the mean velocity v of a "continuous" flow in some cross section x_∞ of the cylindrical capillary at a sufficient distance from the meniscus. One of the formal definitions of D_2 is the following. Let the meniscus be located to the left of the cross section x_∞ and let q be the value of the flow in an arbitrary cross section x to the left of $x_\infty (-\infty < x \leq x_\infty)$. In view of the presence of a meniscus, $q \to 0$ as $x \to -\infty$; therefore, the integral $Q = \int_{-\infty}^{x_\infty} q\,dx$ is finite; it corresponds to the total volume of liquid displaced at a given moment of time with respect to the walls of the capillary to the left of the cross section x_∞. For unperturbed Poiseuille flow with a mean velocity v, this volume of liquid is displaced in a section of finite length $L = Q/\pi R^2 v$. We now determine D_2 as $D_2 \equiv D_P L$, where $D_P = 8\pi\eta_0 v^2$. In practice, v is equal to the experimentally observed velocity of the meniscus. The details of the formal definition of D_2 may be found essential in cases where the level of the Poiseuille dissipation is relatively high, for example, in narrow capillaries and for large velocities.

The values of q and D_1 are found directly by solution of the equations of motion in the section $-\infty < x \leqslant x_\infty$. In the approximation of a flat film, $q = 4\pi R h^3 G/3\eta_0$, and for the models under consideration $D_1 = 2\pi R \int_{-\infty}^{x_\infty} D_f\,dX$, where D_f is determined in accordance with (2.6).

Finding the difference $D = D_1 - D_2$, we can formally define the dynamic resistance of the meniscus as the force F developing the power $D = Fv$. At unit area of the cross section of the capillary, there arrives the additional pressure of the meniscus

$$p = D/\pi R^2 v. \tag{2.8}$$

Calculation of p using formulas (2.2) and (2.8) for a capillary with $R \sim 1$ μm within the framework of a model of an unstable film gives close results. This indicates that both of the proposed methods for determination of p for sufficiently wide capillaries are essentially adequate. In this case the first method must be preferred as simpler. However, the second method is more rigorous and must be preferred for narrower capillaries ($R \leq 0.1$ μm), where the dimensions of the zone of transitional curvature become comparable with the dimension of the meniscus and the use of R_m as a characteristic parameter of the meniscus loses its meaning.

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FINITE-AMPLITUDE INTERNAL WAVES AT AN INTERFACE BETWEEN TWO HEAVY LIQUIDS

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UDC 532.591

The problem of steady-state waves at an interface between two heavy liquids has been discussed in several papers (see, e.g., [1, 2]). Here a method is proposed on the basis of reduction of the problem to the solution of a nonlinear conjugation problem.

Let us consider the flow of two incompressible liquids of different densities in a gravity field with specified velocities at an infinite distance from the interface. We consider the motion to be irrotational and assume that the interface line l, which moves at a certain horizontal velocity U without changing shape, is a Lyapunov curve with period λ. We set up a coordinate system OXY moving in the direction of wave propagation with velocity U. We assume that the absolute particle velocity of the liquid at the interface differs from the wave-propagation velocity. Under this condition the waves are nonbreaking [3].

We place the origin at the average level of the liquid interface line, directing the axis OX along the horizontal in the direction of absolute motion of the line l, and the axis OY along the vertical upward through one of the wave crests (Fig. 1). By Ω_k, k = 1, 2, we denote the domains with period λ occupied by the upper and lower liquids. We introduce the complex variables $Z_k = X_k + iY$ in Ω_k, corresponding to the complex-valued potentials $W_k = \Phi_k + i\Psi_k$ and complex velocities $V_k = dW_k/dZ_k$. We denote the absolute velocities of the liquids at an infinite distance from the interface by $V_{k\infty}$ and the densities by $\rho_k (\rho_1 < \rho_2)$.

We transform to dimensionless variables, putting $V_k = v_k F_{1\infty}$, $Z_k = z_k\lambda/2\pi$, and $W_k = w_k V_{1\infty}\lambda/2\pi$.

Under the stated assumptions the problem reduces to the determination of the wave profile and functions v_k that are analytic in Ω_k and satisfy the kinematic and dynamic conditions at l as well as the following condition at an infinite distance from the interface:

$$\begin{gathered}\psi_1 = \psi_2 = 0 \text{ at } l;\\ \operatorname{Im}(z) = [m_1|v_1(z)|^2 - (1+m_1)|v_2(z)|^2]\,\mathrm{Fr}/2\gamma^2 + c,\ z\in l;\\ v_1 \to 1-\gamma,\ y_1\to\infty;\ v_2\to\delta-\gamma,\ y_2\to-\infty,\end{gathered} \tag{1}$$

where $\mathrm{Fr} = U^2 2\pi/g\lambda$; $m_1 = \rho_1/(\rho_2 - \rho_1)$; $\gamma = U/V_{1\infty}$; $\delta = V_{2\infty}/V_{1\infty}$; g is the acceleration of gravity; and c is a certain functional.

We investigate the auxiliary plane of the complex variable u. Let the domain D^+ be the interior of the unit disk with center at the point u = 0 and D^- the exterior of the disk with

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